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Domain-general cognitive abilities in complex mathematical word problem solving

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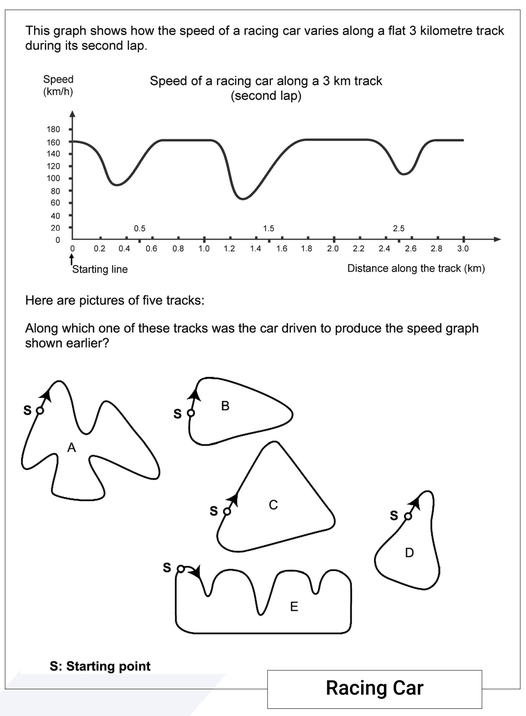
Introduction

Word problems are mathematical tasks in which relevant information is presented as text rather than in mathematical notion (Verschaffel et al., 2010). They require learners to integrate **domain-specific mathematical**, **domain-general linguistic**, and **visuo-spatial abilities**, varying with item characteristics (Boonen et al., 2013; Reinhold et al., 2020). During solution, these abilities are assumed to **interact**—they are typically not applied sequentially, but in parallel (Daroczy et al., 2015).

Fig. 1. *Complex word problems from the PISA mathematics assessment with varying item characteristics.*



Varying item characteristics:
- necessity of calculations
- presence of illustrations
- closeness of answer format
- amount of processable text
- amount of different solution heuristics
- etc.

We addressed the relative importance of different individual domain-general characteristics that may contribute to word problem solving by including both the individual level and the item level. We focused on **Complex Word Problems** (CWPs) which combine **multiple forms of representations**, **irrelevant information**, **notable amounts of text**, and **functional real-world contexts** (Strohmaier et al., 2021).

Method

We investigated the effects of verbal, numerical, spatial, and general reasoning abilities on CWP-performance among ***N* = 1,282 first-year university engineering students**.

CWP-solving ability was assessed with six items from the pool of published **PISA mathematics items**. Spatial ability was measured with the **Mental Rotations Test**; Verbal ability with a **verbal analogies scale**; Numerical ability with a **calculations scale**; and General reasoning ability with a short form of **Raven's Advanced Progressive Matrices**.

Results & Discussion

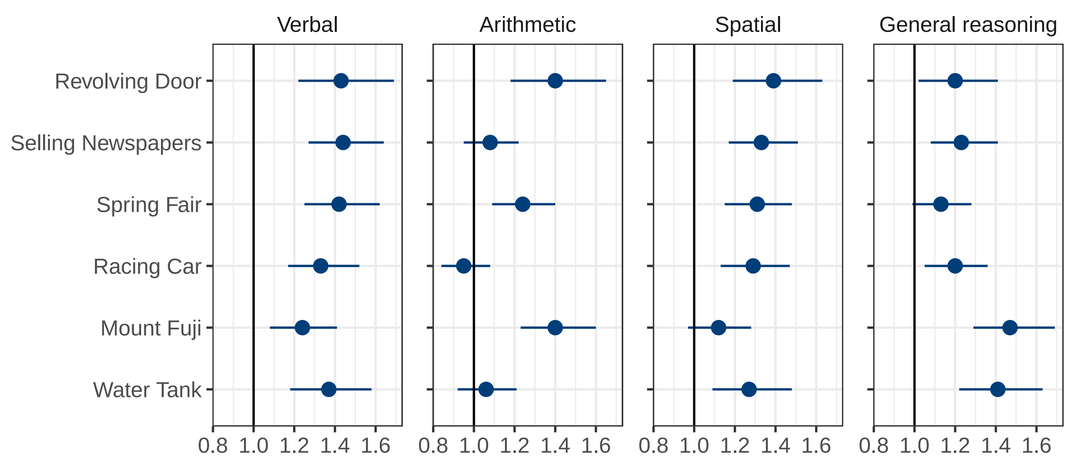
The effect of domain-general abilities differed between the items. The model-fit comparison showed that the model including the **cognitive skills × task interaction** fitted the data

Tab. 1. *Characteristics and correlations of the cognitive skills scales.*

	Descriptive results						Correlations		
Cognitive skills	α	Range	Min.	Max.	*M*	*SD*	*1.*	*2.*	*3.*
1. Verbal	.65	0–20	0	19	12.16	3.11	—		
2. Arithmetic	.82	0–20	0	20	15.39	3.65	.22	—	
3. Spatial	.91	0–24	0	24	11.92	6.07	.29	.22	—
4. General reasoning	.67	0–12	0	12	8.35	2.42	.28	.30	.30

significantly better than the model including only main effects, $X^2 = 48.54$, $\Delta AIC = 8.54$, $p < .001$.

Fig. 2. *Odds ratios and their 95 % confidence intervals of cognitive skills (+ 1 SD) on the solution probability of six different complex word problems, in descending order of task difficulty.*



The influence of numerical ability was strongest for items requiring a numerical answer based on **calculations**. For spatial ability, all items that **contained illustrations** showed a similar influence of spatial ability. Yet, in our data there was no difference between items that addressed the content area of geometrical shapes—and items that used pictures only to illustrate mathematical problems. General reasoning ability has shown beneficial in items that allowed for **generic problem-solving heuristics** not necessarily related to mathematics. Verbal ability was the only predictor that influenced the solution rate of all items significantly, underlining the **close association between language and mathematics** that emerges in CWP-solving (Daroczy et al., 2015; Reinhold et al., 2020).

Our results underpin that CWP-solving requires a broad facet of domain-general cognitive abilities besides mere domain-specific mathematical competencies—and that their influence differs notably between items (Strohmaier et al., 2021).

References

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