

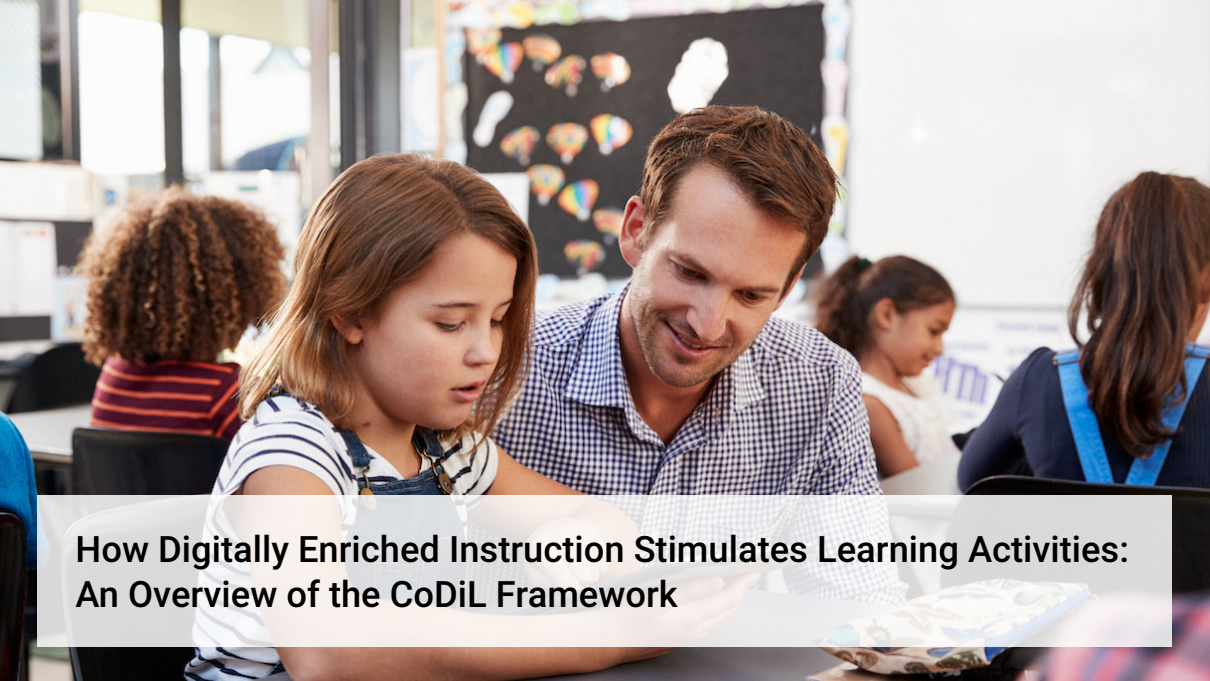


Understanding How Digital Tools Foster Mathematical Learning: Framework-Guided Investigations in Primary and Secondary Classrooms

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How Digitally Enriched Instruction Stimulates Learning Activities: An Overview of the CoDiL Framework

Introduction

Di.ge.LL

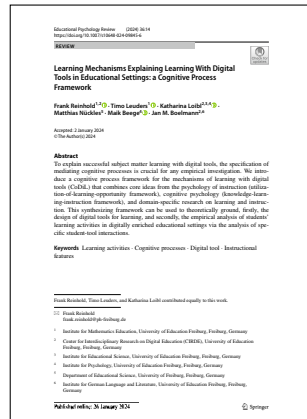
Research Training Group: Digitally-supported teaching-and-learning environments for cognitive activation.

- **Objectives:** Development and empirical validation of research-based teaching-learning methods with digital tools for use in the classroom.
- **Research questions:** Stimulating cognitive activation in phases of learning new content.

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Reinhold, Leuders, Loibl, Nückles, Beege, & Boelmann. (2024). *Educ. Psychol. Rev.*

A Central Part of my Habilitation Thesis

The CoDiL Framework

Digitally-enriched settings provide instructional features that may be...

- Utilized by students during interaction with the tool which than...

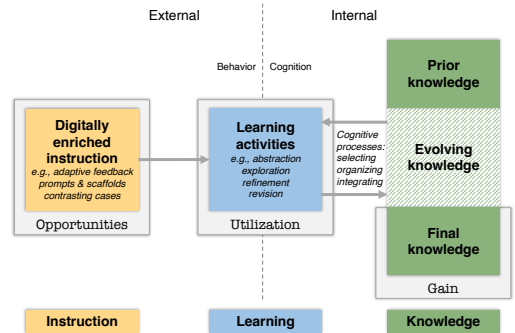
(Utilization-of-Learning-Opportunities Framework by, e.g., Seidel, 2014)

- Stimulate behavioral (observable) and cognitive (latent) learning activities which...

(Evidence-Centered Design Framework by, e.g., Goldhammer et al., 2021)

- Interact with prior knowledge to construct new (domain-specific) knowledge components.

(Knowledge-Learning-Instruction Framework by Koedinger et al., 2012)



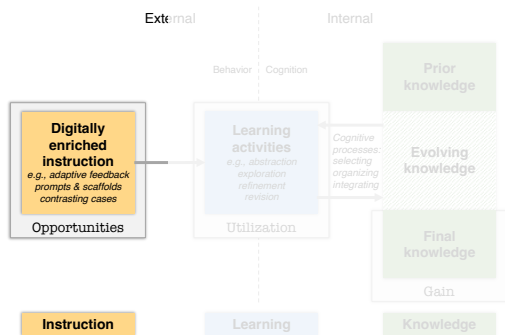
Reinhold, Leuders, Loibl, Nückles, Beege, & Boelmann. (2024). *Educ. Psychol. Rev.*

Development of Digital Learning Environments

Instruction

Aim: Evidence-based designs for digitally enriched learning environments that...

- Incorporate instructional features shaped by affordances and constraints, which...
(Greeno, 1998; Hartson, 2003; Norman, 1999; Scheiter, 2021)
- Focus established mechanisms of teaching and learning to stimulate learning processes...
(Mayer, 2014; Schumacher & Stern, 2023; Sweller, 2020)
- With respect to the new-to-learn domain-specific knowledge components.
(Anderson et al., 1997; Kintsch, 1991; Koedinger et al., 2012; Ritter et al., 2007)



Analyzing Learning Processes in Digital Contexts

Learning

Aim: Fine-grained insights into students' actual learning activities by...

- Considering learning an active and generative process subject to self-regulation, and...

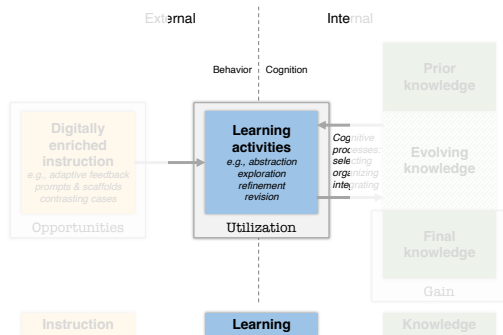
(Fiorella & Mayer, 2015; Fredricks et al., 2004; Molenaar et al., 2023)

- Establishing theory-based links between on-task behavior and cognition while...

(Goldhammer et al., 2017, 2021; Mislevy et al., 2012)

- Combining generic learning analytics with domain-specific process data.

(Greiff et al., 2015; Huber & Bannert, 2023; Reinhold, Strohmaier et al., 2020)



Teaching and Learning in Competence Areas

Knowlegde

Aim: Understand learner-specific but systematic patterns of reasoning and difficulty by...

- Analyzing relevant 21st century skills in relation to subject-specific demands, ...

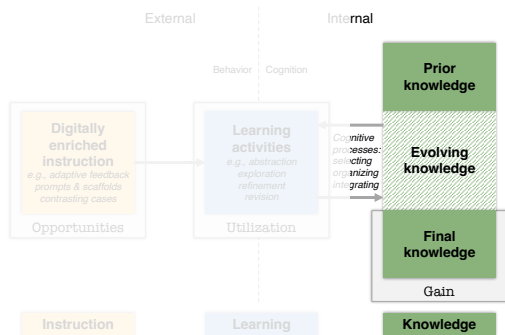
(Bescherer et al., 2024; Martin, 2018; OECD, 2023)


- Connecting educational psychology and subject-matter didactics to...

(Anderson et al., 1997; Kintsch, 1991; Koedinger et al., 2012; Ritter et al., 2007)

- Informing the design of digital tools through content-specific instructional insights.

(Reinhold et al., 2024; Scheiter et al., 2022)



A photograph of a diverse group of Grade 6 students in a classroom. In the foreground, a young girl with curly hair and a plaid shirt is smiling while holding a tablet. Next to her, a boy in a dark blue polo shirt is looking at the camera. Behind them, several other students are seated at desks, some using tablets. The classroom background includes a blue bulletin board with a drawing of a tree, a whiteboard, and shelves with various items.

Teaching *fr*ACTiONs in Grade 6 Classrooms: Focusing Motivated Action in Learning Fractions with Digital Tools

Introduction

The frACTIONS Project

Motivated Action in Learning Fractions with Digital Tools. Do adaptive features of digital learning environments...

- enhance engagement with learning activities...
- during training in mathematics classrooms...
- which in turn mediates learning gains?

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Oppmann, Baege, & Reinhold, (2025).
Learn. Instr.

Introduction

Fractions are notoriously difficult to learn

Conceptual challenge: Understanding fraction equivalence

(Behr et al., 1983; Kieren, 1983; Pedersen & Bjerre, 2021)

Aufgabe 1 Kürze so weit wie möglich.

a) $\frac{18}{72} =$ $\frac{18 : 3}{72 : 3} = \frac{6}{24}$

b) $\frac{21}{63} =$ $\frac{21 : 7}{63 : 7} = \frac{3}{9}$

c) $\frac{7}{7} =$ $\frac{7 : 7}{7 : 7} = \frac{1}{1}$

Student from German „Hauptschule“, grade 6, after 16 lessons of fractions instruction

Fractions are notoriously difficult to learn

Conceptual challenge: Understanding fraction equivalence

(Behr et al., 1983; Kieren, 1983; Pedersen & Bjerre, 2021)

Aufgabe 7 Ergänze die fehlenden Zahlen in den Kästen.

a) $\frac{3}{\boxed{4}} = \frac{12}{28}$

b) $\frac{6}{8} = \frac{\boxed{2}}{20}$

c) $4 = \frac{\boxed{1}}{3}$

Student from German „Gymnasium“, grade 6, after 16 lessons of fractions instruction

Introduction

Fractions are notoriously difficult to learn

Conceptual challenge: Understanding fraction equivalence

(Behr et al., 1983; Kieren, 1983; Pedersen & Bjerre, 2021)

- Involves typical hurdles and misconceptions
 - Overgeneralization of natural number concepts, e.g., „unique representations“
 - Misinterpretation of everyday terms, e.g., „reducing should make smaller“
 - Problems with the algorithmic procedure, e.g., $\frac{5}{3} = \frac{5+3}{3+3} = \frac{8}{6}$, or $\frac{14}{45} = \frac{1}{5}$

(Eichelmann et al., 2012; Obersteiner et al., 2015; Prediger, 2008; Reinhold, Hoch et al., 2020)

- Importance of fostering conceptual ideas before procedural training
- Expanding & reducing fractions by refining & coarsening part-whole representations

(Lamon, 2020; Reinhold, Hoch et al., 2020)

Introduction

Features of educational technology can support cognitive learning processes

Instructional challenge: Learning opportunities need to be taken to be effective

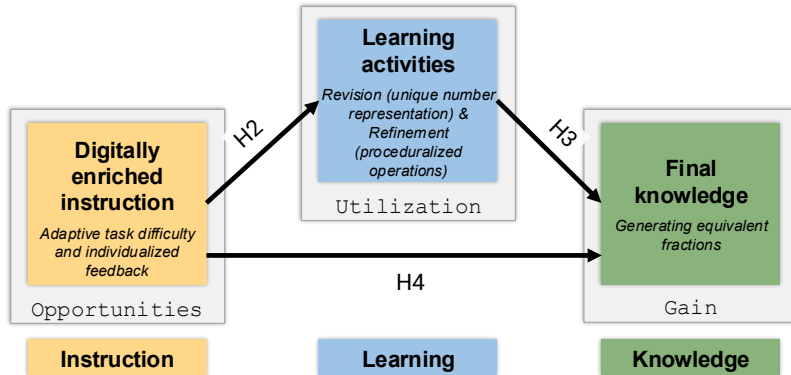
(Chi & Wylie, 2014; Reinhold et al., 2024; Seidel, 2014)

- Adaptive task difficulty can prevent cognitive overload
(Kalyuga & Sweller, 2005; Moreno & Mayer, 2007; Sweller, 2020)
- Corrective and explanatory feedback can reduce extraneous load
(Hattie & Timperley, 2007; Reinhold et al., 2024; Wisniewski et al., 2020)
- But the effectiveness of these features in technology-based learning may depend on ...
 - Prior knowledge
(Demedts et al., 2024)
 - Behavioral and cognitive engagement
(Fredricks et al., 2004; Goldhammer et al., 2021; Reinhold, Strohmaier et al., 2020)

Research Questions and Hypotheses

Study 1: The frACTIONS Project

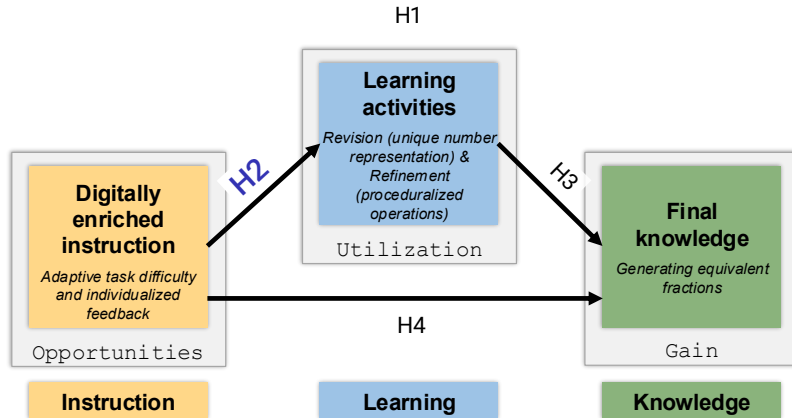
H1



Hypothesis 1:
Students differ in how they engage in the provided training material during mathematics lessons.

Research Questions and Hypotheses

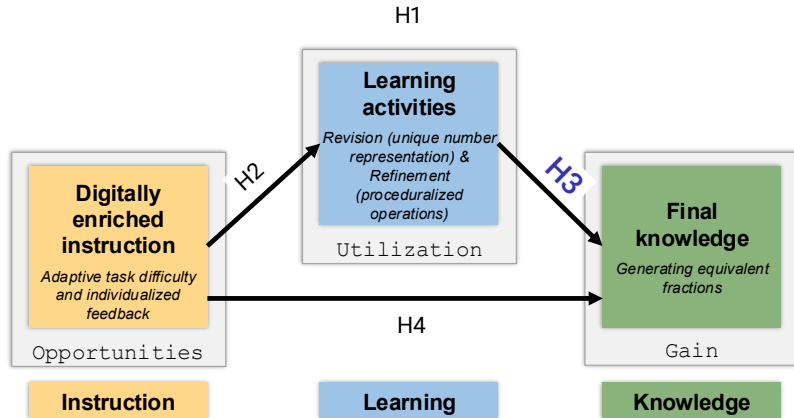
Study 1: The frACTIONS Project



Hypothesis 2:
Adaptivity and feedback influence how students engage in the provided training material.

Research Questions and Hypotheses

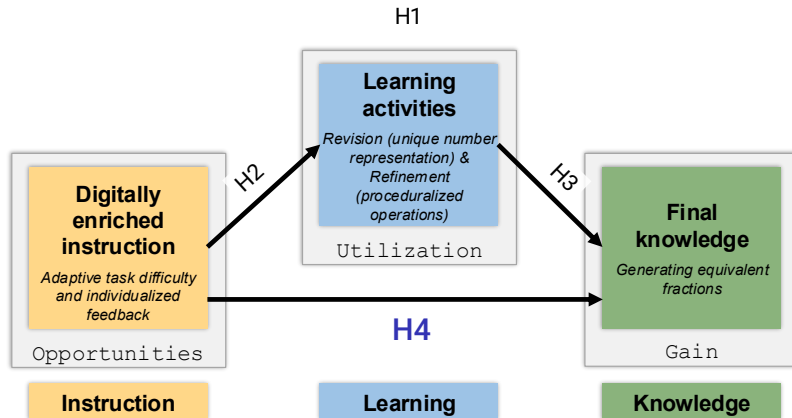
Study 1: The frACTIONS Project



Hypothesis 3:
Different engagement with the provided training material leads to differences in content-specific knowledge after training.

Research Questions and Hypotheses

Study 1: The frACTIONS Project

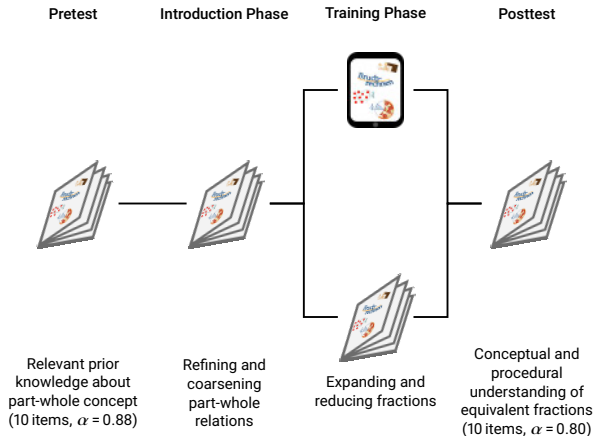


Hypothesis 4:
Adaptivity and feedback have a positive effect on students' content-specific knowledge after training—mediated by engagement.

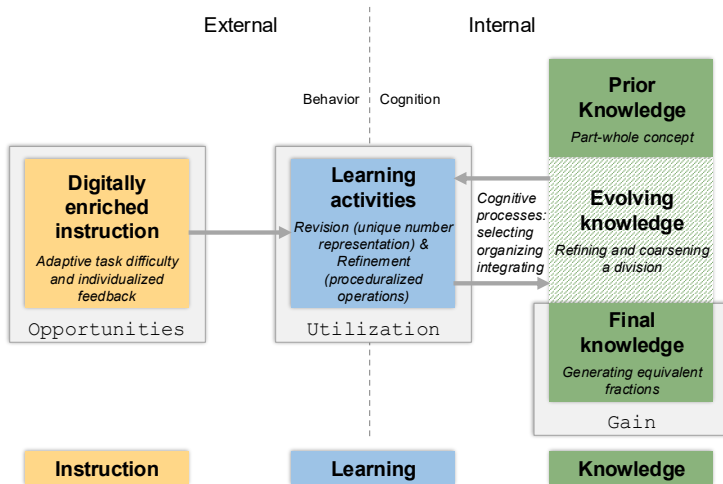
Method

Study Design

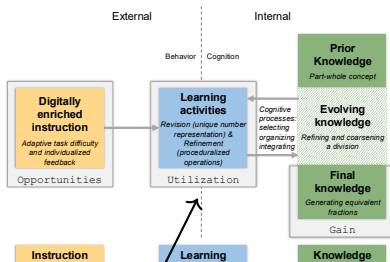
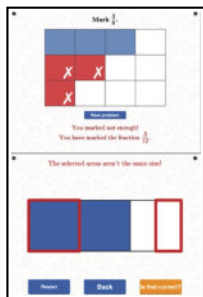
- **Sample:** Randomized controlled trial with $N = 300$ 6th-grade students
- **Experimental Group (EG):** Adaptivity & feedback (Digitally-enriched ITS)
- **Control Group (CG):** Linear increasing difficulty & no feedback (Paper-based)
- 90 minutes intervention during regular mathematics instruction at school



Method



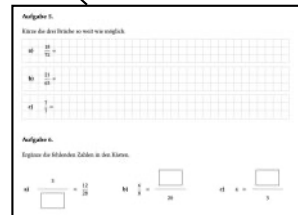
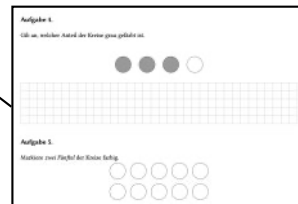
Method



Linking on-task behavior to learning activities.

Derived process measures during training phase:

- Number of worked-on tasks in each difficulty cluster
- Solution rate in each difficulty cluster



Results

Hypothesis 1: Different engagement with training material

Gaming the System



only engaging with low levels of difficulty · low success rates · **pushing through large amount of tasks**

The Average



moderate success rates · low number of worked-on tasks · mostly tasks of medium difficulty

Accelerated Experts



skipping low levels of difficulty · high success rates · most worked-on tasks on highest difficulty

Accelerated Workers



high success rate in easiest tasks · moderate success in medium tasks · not reaching highest difficulty

Slowed-down Experts



trapped in low levels of difficulty · high success rates · only few to no tasks of highest difficulty

Industrious Experts



high success rates · medium number of worked-on tasks · barely reaching highest difficulty levels

k-Means Cluster Analysis of 14 metric process indicators per student—derived from 11,071 observations

Results

Hypothesis 2: Features influence engagement

Gaming the System



The Average



Accelerated Experts



Accelerated Workers



Slowed-down Experts



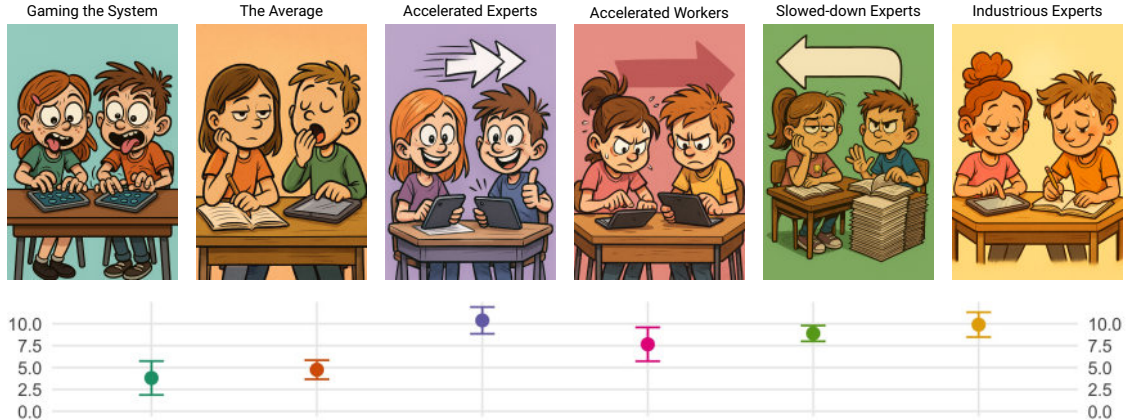
Industrious Experts



Multinomial logistic regression: $X^2(5) = 96.68, p < 0.001$

Results

Hypothesis 3: Different engagement leads to different posttest outcomes



One-way ANOVA: $F(5, 257) = 14.45, p < 0.001, \eta_p^2 = 0.22$

Results

Hypothesis 4: Engagement mediates success of adaptive features

- Welsh *t*-Test.

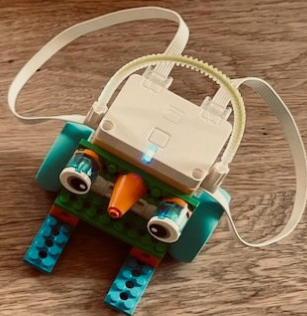
Scale	Control		Experimental		<i>t</i>	<i>df</i>	<i>p</i>	<i>d</i>
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>				
Posttest Outcome	7.172	4.884	7.295	5.009	-0.216	297.555	0.830	0.025

But: Both low-engaged worst performing subgroup (**Gaming the system**), and high-engaged best performing subgroup (**Accelerated Experts**) overrepresented in EG

- Structural equation model. Significant indirect path in line with hypothesis:

$$\beta = -0.173^*$$

$$CFI = 0.992, TLI = 0.991, RMSEA = 0.023$$



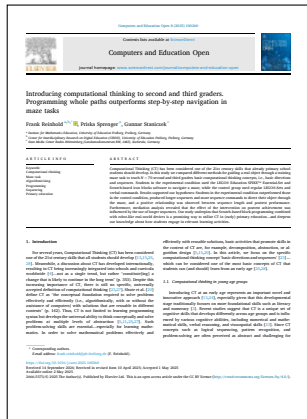
**Teaching *Computational Thinking* in Grade 2 and 3 Classrooms:
LEGO®-Enriched Programming in Mathematics**

Introduction

The COMPI.LE Project

Understanding learning processes in early CT education.

- Investigates how **engagement with CT activities** can be enhanced through age-appropriate instructional design.
- Focuses on **primary school mathematics classrooms** using block-based programming and tangible tools.
- Addresses the open question of **how underlying learning processes unfold** during early CT instruction.



What is Computational Thinking

- Computational Thinking (CT) is considered one of the **21st-century skills** that all students should develop

(Manches & Plowman, 2017; OECD, 2023; Wing, 2006; Yildirim & Uluyol, 2023)

- CT is increasingly being **integrated into schools and curricula internationally**

(Broley et al., 2024)

- No universally accepted definition; however: *„the conceptual foundation required to solve problems effectively and efficiently (i.e., algorithmically, with or without the assistance of computers) with solutions that are reusable in different contexts“*

(Shute et al., 2017; see also Tsarava et al., 2022; Yeni et al., 2023)

- CT supports the ability to think **abstractly** and to **solve problems**

(Büscher, 2024; Lee & Lee, 2024; Wing, 2006; Yeni et al., 2023)

Activities and Processes in the Context of CT

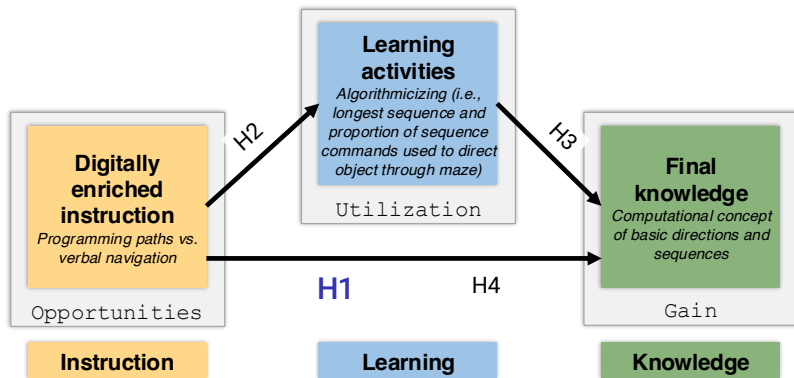
- A qualitative study with 11- to 13-year-olds using 'Turtlecoder' revealed differences in activities such as:
 - Reusing and modifying code
 - Decomposing problems
 - Testing and evaluating solutions
 - **Algorithmic thinking**
 - Abstraction
 - Generalization
- (Büscher, 2024; see also Brennan & Resnick, 2013; Selby & Woollard, 2013)
- Those differences in the problem-solving processes among 11- to 13-year-olds suggest that these processes might be even more heterogeneous in **younger students**.

Foundational Concepts for Promoting CT

- A central concept for the early development of CT is the *„ability to formulate and solve problems by relying on the fundamental concepts of computing, and using logic-syntax of programming languages“*
(Román-González et al., 2017)
- In assessment contexts, often operationalized as **basic directions and sequences**
(Román-González et al., 2017; Shute et al., 2017; Tank et al., 2024; Tsarava et al., 2022; Zapata-Caceres et al., 2020; Zhang & Wong, 2024)
- Plausible starting point for CT interventions with young learners:
 - Focus: **Algorithmic thinking** as explicit articulation of at least two steps in a sequence
(Büscher, 2024)
 - Implemented in block-based digital CT learning settings or in „unplugged“ CT activities
(Ersozlu et al., 2023; Tank et al., 2024; Zhang & Wong, 2024)

Research Questions and Hypotheses

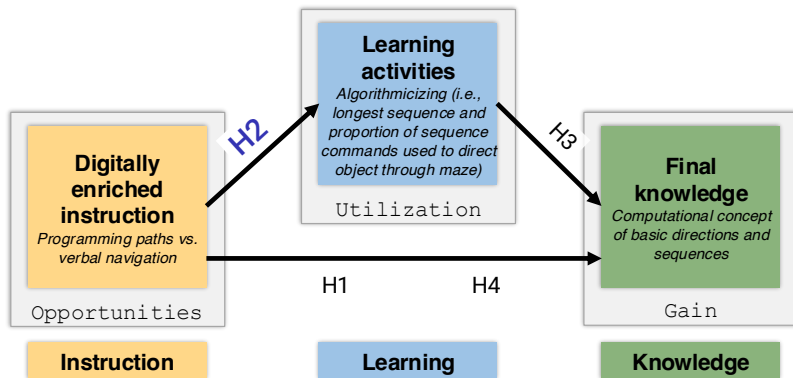
Study 2: The COMPI.LE Project



Hypothesis 1:
Programming paths in maze tasks outperforms unplugged verbal navigation when learning the computational concept of basic directions and sequences for the first time.

Research Questions and Hypotheses

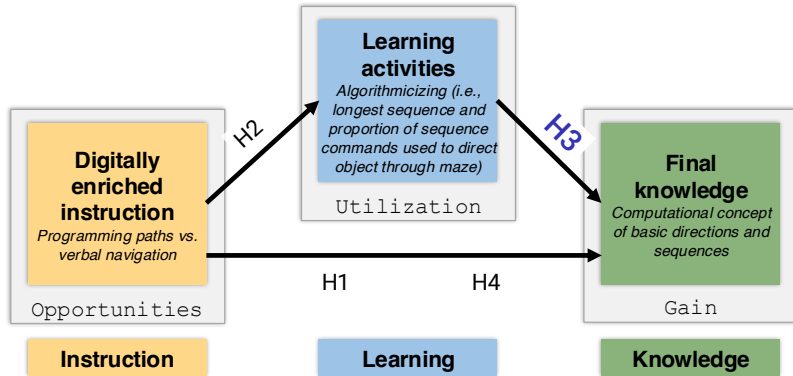
Study 2: The COMPI.LE Project



Hypothesis 2:
Programming paths
(vs. unplugged verbal
navigation) influences
how algorithmicizing
processes are stimu-
lated during learning.

Research Questions and Hypotheses

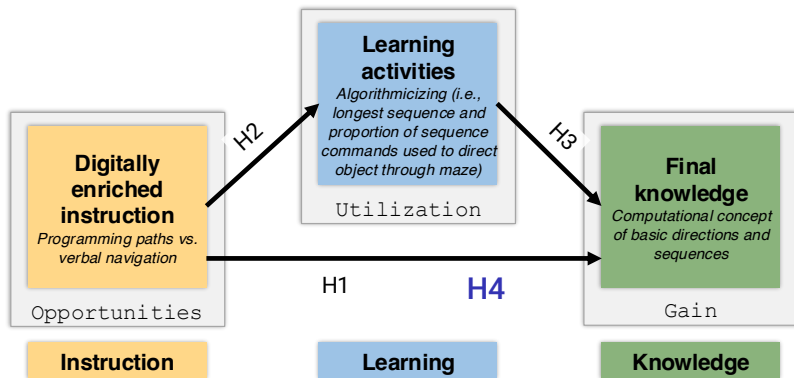
Study 2: The COMPI.LE Project



Hypothesis 3:
Different Algorithmicizing processes lead to differences in content-specific knowledge after training.

Research Questions and Hypotheses

Study 2: The COMPI.LE Project

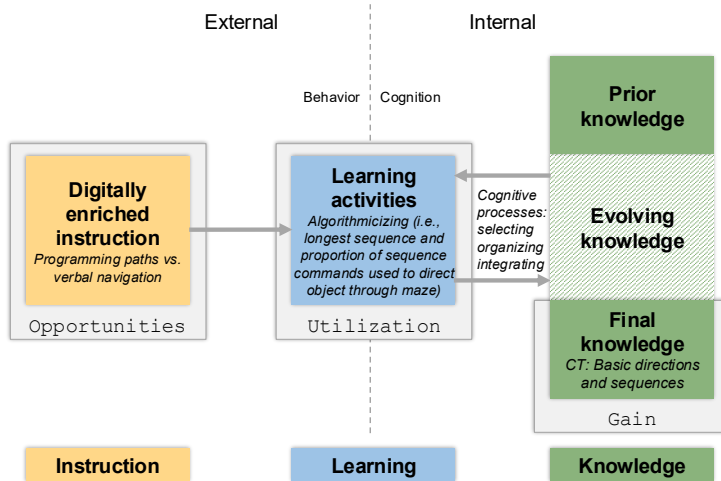


Hypothesis 4:
The effect of programming whole paths vs. verbal navigation is mediated by algorithmicizing processes during learning.

Study Design

- Randomized controlled trial with $N = 70$ 6th- and 7th-grade students in pairs
- **Procedure:** 5 min (Pretest), 30 min (Intervention), 10 min (Posttest)
- **Intervention:** Assembly phase and Labyrinth phase; story-based navigation through a complex arctic maze (19 commands necessary)
 - **EG:** LEGO® SPIKE™ Essential with Scratch-based Icon-Block programming
(e.g., Ersozlu et al., 2023; Zhang & Wong, 2024)
 - **CG:** Regular LEGO®-Set with „unplugged“ verbal control commands
(e.g., Hsu et al., 2018)
- **Process data analysis:** Videography of all sessions; coding of all commands, the sequence commands, and the length of the sequence commands

Method



Method



Fig. 2a: Experimental condition (front view)



Fig. 2b: Control condition (front view)



Fig. 2c: Experimental condition (side view)

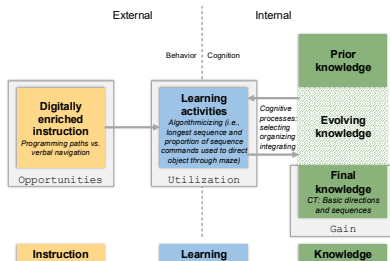


Fig. 3a: Instructional item



Fig. 3d: Posttest item 3



Fig. 3b: Pretest item 1 and Posttest item 1



Fig. 3c: Posttest item 4



Fig. 3c: Posttest item 2

Pac-Man labyrinth tasks

(e.g., Tsarava et al., 2022; Zapata-Cáceres et al., 2020)

Hypothesis 1: Different engagement with training material

	Model 0		Model 1		Model 2		Model 3	
<i>Fixed effects</i>	<i>OR (SE)</i>	<i>p</i>	<i>OR (SE)</i>	<i>p</i>	<i>OR (SE)</i>	<i>p</i>	<i>OR (SE)</i>	<i>p</i>
Prior knowledge	—	—	2.68 (1.25)	0.034	3.22 (1.49)	0.011	3.79 (1.71)	0.003
Grade level [2 → 3]	—	—	—	—	3.92 (1.74)	0.002	3.82 (1.63)	0.002
Condition [Ctr. → Exp.]	—	—	—	—	—	—	3.36 (1.42)	0.004
<i>Random effects</i>	<i>Var</i>	<i>PCV</i>	<i>Var</i>	<i>PCV</i>	<i>Var</i>	<i>PCV</i>	<i>Var</i>	<i>PCV</i>
Student ($N = 70$)	1.96	—	1.79	8.7%	1.39	22.3%	1.10	20.8%
Item ($k = 4$)	0.80	—	0.80	—	0.81	—	0.82	—
<i>Model characteristics</i>	<i>Estimate</i>	<i>p</i>	<i>Estimate</i>	<i>p</i>	<i>Estimate</i>	<i>p</i>	<i>Estimate</i>	<i>p</i>
R^2 (marginal)	0.000	—	0.038	—	0.114	—	0.172	—
R^2 (conditional)	0.456	—	0.462	—	0.469	—	0.478	—
AIC	340.905	—	338.356	—	330.309	—	323.538	—
χ^2	—	—	4.549	0.033	10.047	0.002	8.772	0.003

Hypothesis 2: Features influence engagement

<i>Predictors</i>	Longest sequence		Relative number of sequences	
	<i>IRR (SE)</i>	<i>p</i>	<i>Est. (SE)</i>	<i>p</i>
Prior knowledge	0.80 (0.12)	0.144	0.93 (0.45)	0.874
Grade level [2 → 3]	2.35 (0.34)	<.001	2.21 (0.98)	0.073
Condition [Ctr. → Exp.]	8.96 (2.00)	<.001	4.80 (2.12)	<.001

Hypothesis 3: Different engagement leads to different posttest outcomes

Prediction of solution probabilities for items in the posttest:

Longest sequence: $OR = 2.20^{***}$ 95% CI [1.36, 3.55]

Relation of sequence commands: $OR = 1.95^{**}$ 95% CI [1.22, 3.14]

Hypothesis 4: Engagement mediates success of digital features

Indirect Paths in Mediation Model:

Longest sequence: $\beta = 0.614^{**}$

Relation of sequence commands: $\beta = 0.053^{ns}$



**Investigating the Role of Digital Tools in Fostering Learning:
Insights from CoDiL-Studies in Mathematics Classrooms**

Discussion: Study 1 (The frACTIONs Project)

Implications

- Engagement with learning activities is **essential** to realizing the potential of adaptive instructional features.
- Scaffolds worked for high-achieving students—but **distracted low-achieving students** and even hindered their learning.
- **Data-driven classification** of student engagement patterns offers a promising basis for **tailoring instruction**.

Limitations and Future Directions

- Control group received **non-digital instruction**—limiting conclusions about adaptivity effects per se.
- **Short duration** of the intervention may have been insufficient to produce measurable learning effects.
- Future work should explore **long-term implementations** and **disentangle affordances** of adaptivity and feedback.

Discussion: Study 2 (The COMPI.LE Project)

Implications

- Findings **extend prior research on CT instruction**, demonstrating that CT integration is feasible in primary school.
- **Block-based programming** outperformed unplugged instruction—due to more engagement in algorithmic thinking.
- The study demonstrates how **CT can be meaningfully integrated** early within regular math instruction.

Limitations and Future Directions

- **Small sample size** limits statistical power and generalizability—but keep large effects in mind.
- **Short intervention** limits insight into long-term learning and development of CT skills.
- Future work should **broaden CT content** beyond basic sequences and directions and explore age-specific support needs.

Thank you for your attention.

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K. Loibl



M. Nückles



J. M. Boelmann



Study 1: frACTIONS



M. Beege



M.-M. Oppmann



Study 2: COMPILE



P. Sprenger



G. Staniczek